Alexei Karlovich
Universidade Nova de Lisboa
and
Centro de Análise Funcional e Aplicações, IST-UT Lisbon, Portugal
oyk@fct.unl.pt

The Cauchy singular integral operator on weighted variable Lebesgue spaces

Let $p : \mathbb{R} \to (1, \infty)$ be a globally log-Hölder continuous variable exponent and $w : \mathbb{R} \to [0, \infty]$ be a weight. We prove that the Cauchy singular integral operator S is bounded on the weighted variable Lebesgue space $L^{p(\cdot)}(\mathbb{R}, w) = \{f : fw \in L^{p(\cdot)}(\mathbb{R})\}$ if and only if the weight w satisfies

$$\sup_{-\infty<a<b<\infty} \frac{1}{b-a} \|w\chi_{(a,b)}\|_{p(\cdot)} \|w^{-1}\chi_{(a,b)}\|_{p'(\cdot)} < \infty \quad (1/p(x) + 1/p'(x) = 1).$$

This is a joint work with Ilya Spitkovsky.